Oppgaver fra 1.7

183, 184, 185, 186, 189, 194

183 c)

$(x-(-2))^2+(y-3)^2+(z-(-1))^2 = 6^2 \Leftrightarrow$
$(x+2)^2+(y-3)^2+(z+1)^2 = 6^2$

(Ikke noe poeng i å ekspandere til: $x^2+4x+y^2-6y+z^2+2z+14-22 = 0$)

184 c)

$x^2+6x+3^2+y^2-y+(\frac{1}{2})^2+z^2-14z+7^2 = 2+3^2+(\frac{1}{2})^2+7^2$
$(x+3)^2+(y-\frac{1}{2})^2+(z-7)^2 = (\frac{\sqrt{241}}{2})^2$

Sentrum: $S = (-3, \frac{1}{2}, 7)$ Radius: $R = \frac{\sqrt{241}}{2} \approx 7.76$

185

$x^2-2x+1^2+y^2+6y+3^2+z^2+2z+1^2 = 5+1^2+3^2+1^2 \Leftrightarrow$
$(x-1)^2+(y+3)^2+(z+1)^2 = 4^2$):$S_1 = (1,-3,-1)$, $R_1 = 4$

$x^2+8x+4^2+y^2-4y+2^2+z^2-2z+1^2 = 15+4^2+2^2+1^2 \Leftrightarrow$
$(x+4)^2+(y-2)^2+(z-1)^2 = 6^2$):$S_2 = (-4,2,1)$, $R_2 = 6$
Avstand mellom sentrene:
$a = \left|\overrightarrow{S_1S_2}\right| = |[-5,5,2]| = \sqrt{5^2+5^2+2^2} = 3\sqrt{6}$

Største avstand: $R_1+a+R_2 = 4+3\sqrt{6}+6 = 3\sqrt{6}+10 \approx 17.3$

186

a)

6 flater, som er parrvis like store:

$A_{uv} = \left|\vec{u}\times\vec{v}\right| = |[3,-1,0]\times[5,1,2]| = |[-2,-6,8]| = \sqrt{2^2+6^2+8^2} = 2\sqrt{26}$

$Avw = \left|\vec{v}\times\vec{w}\right| = |[5,1,2]\times[0,4,6]| = |[-2,-30,20]| = \sqrt{2^2+30^2+20^2} = 2\sqrt{326}$

$A_{wu} = \left|\vec{v}\times\vec{w}\right| = |[0,4,6]\times[3,-1,0]| = |[6,18,-12]| = \sqrt{6^2+18^2+12^2} = 6\sqrt{14}$

Samlet overflate:
$O = 2A_{uv}+2A_{vw}+2A_{wu} = 2(2\sqrt{26})+2(2\sqrt{326})+2(6\sqrt{14}) =$
$12\sqrt{14}+4\sqrt{26}+4\sqrt{326} \approx 138$

b)

$V = \left|(\vec{u}\times\vec{v})\cdot\vec{w}\right| = |[-2,-6,8]\cdot[0,4,6]| = 24$

189

Fire avgrensende plan:

$$\alpha : x = 0$$
$$\beta : y = 0$$
$$\gamma : z = 0$$
$$\delta : 2x - y + 4z - 4 = 0$$

a)

Skjæring mellom α, β, γ: $O = (0,0,0)$
Skjæring mellom β, γ, δ: $2x - 0 + 4 \cdot 0 - 4 = 0 \Leftrightarrow x = 2$): $A = (2,0,0)$
Skjæring mellom α, γ, δ: $2 \cdot 0 - y + 4 \cdot 0 - 4 = 0 \Leftrightarrow y = -4$): $B = (0,-4,0)$
Skjæring mellom α, β, δ: $2 \cdot 0 - 0 + 4z - 4 = 0 \Leftrightarrow z = 1$): $C = (0,0,1)$

b)

$$V = \frac{1}{6}\left|(\overrightarrow{OA} \times \overrightarrow{OB}) \cdot \overrightarrow{OC}\right| = \frac{1}{6}|([2,0,0] \times [0,-4,0]) \cdot [0,0,1]| =$$
$$\frac{1}{6}|[0,0,-8] \cdot [0,0,1]| = \frac{8}{3} = \frac{4}{3} \approx 1.33$$

c)

$$A = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}|[-2,-4,0] \times [-2,0,1]| = \frac{1}{2}|[-4,2,-8]| = |[-2,1,-4]| =$$
$$\sqrt{2^2 + 1^2 + 4^2} = \sqrt{21} \approx 4.58$$

Normalvektor til største sideflate: $\overrightarrow{AB} \times \overrightarrow{AC} = [-4,2,-8] = -2[2,-1,4]$
Velger: $\vec{n} = [2,-1,4]$

Normalvektor til xy –planet: $\vec{e_z} = [0,0,1]$

Vinkel mellom plan:
$$\cos\theta = \frac{\vec{e_z} \cdot \vec{n}}{|\vec{e_z}||\vec{n}|} = \frac{[0,0,1] \cdot [2,-1,4]}{1\sqrt{21}} = \frac{4}{\sqrt{21}} \Rightarrow \theta \approx 29.2^\circ$$

194

a)

$\overrightarrow{AB} = [-2,4,0]$, $\overrightarrow{AC} = [-1,0,3]$

Lager normalvektor: $\overrightarrow{AB} \times \overrightarrow{AC} = [-2,4,0] \times [-1,0,3] = [12,6,4] = 2[6,3,2]$
Velger: $\vec{n} = [6,3,2]$

Bruker punktet A og lager ligning: $[x-5,y,z] \cdot \vec{n} = 0 \Leftrightarrow$
$[x-5,y,z] \cdot [6,3,2] = 0 \Leftrightarrow 6x + 3y + 2z - 30 = 0$

Avstand fra Origo: Generelt: Planet $ax + by + cz + d = 0$ vil ha denne avstanden fra Origo:
$\frac{|a \cdot 0 + b0 + c \cdot 0 + d|}{\sqrt{a^2+b^2+c^2}} = \frac{|d|}{\sqrt{a^2+b^2+c^2}}$ Et resultat det kan være verdt å merke seg!

Her får vi avstanden: $s = \frac{|-30|}{\sqrt{6^2+3^2+2^2}} = \frac{30}{7} \approx 4.29$

b)

Regner raskt ut at $OA = OB = OC = R = 5$

Kuleflaten blir da: $x^2 + y^2 + z^2 = 5^2$

c)

Se figurer side 66!

Høyden i kulesegmentet: $h = R - s = 5 - \frac{30}{7} = \frac{5}{7}$
Radien i kulesegmentet: $r^2 = R^2 - s^2 = 5^2 - (\frac{30}{7})^2 = \frac{325}{49}$

Overflaten i kulesegmentet:

$$O = 2\pi Rh + \pi r^2 = 2\pi 5 \cdot \frac{5}{7} + \pi \frac{325}{49} = \frac{675}{49}\pi \approx 43.3$$